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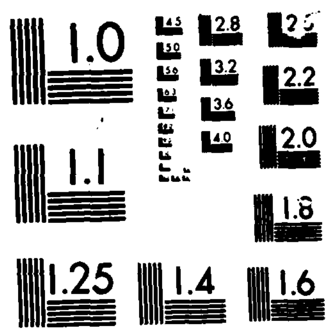

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ONE-COMPONENT PLASMA MODEL AND LARGE Z LABORATORY PLASMAS

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Stemming from an expression for the mean two-particle potential energy of a two-component laboratory plasma comprised of ions of charge Ze and electrons, it is argued that for sufficiently large Z, thermodynamic properties of such plasmas are the same as those relevant to Wigner's one-component plasma model. Thus, an equation of state is obtained for a strongly coupled laboratory plasma with $Z \geq 5$.

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INTRODUCTION

Stemming from an expression for the mean two-particle potential energy of a two-component laboratory plasma comprised of ions of charge Ze and electrons, it is argued that for sufficiently large Z, thermodynamic properties of such plasmas are the same as those relevant to Wigner's one-component plasma model. Thus the following equation of state is obtained for a laboratory plasma with $Z \gtrsim 5$ and $\Gamma \geq 1$.

$$\frac{P}{nk_BT} = 1 + \frac{d}{3} + \frac{1}{3}\left(\bar{a}\Gamma + b\Gamma^{1/4} + c\Gamma^{-1/4}\right)$$

In this equation, P is pressure, T is temperature, n is ion number density and $\bar{a}$, b, c, d are known numerical constants. The plasma parameter $\Gamma = (Ze)^2/ak_BT$ where $4\pi a^3/3$ represents mean occupation volume per ion.

DISCUSSION

Strongly coupled plasmas play a role in recombination approaches to x-ray lasing [1], inertial-confinement fusion devices [2], the interiors of certain super-dense stars [3], and in plasma-driven rail-gun devices [4].

For the most part studies addressing strongly coupled plasmas [5,6,7] have employed a plasma model due to Wigner [8] which was conceived for the purposes of studying phase transition to the solid state. This medium is called "jellium", or more commonly, a "one-component plasma", which often carries the abbreviation, OCP. An OCP is comprised of ions moving in a charge-neutralizing uniform negative background.

In the present work attention is directed at a laboratory plasma comprised of ions of charge Ze and electrons in equilibrium at a given temperature. Examining the interaction energy of the plasma indicates that for $Z \gtrsim 5$, thermodynamic properties such as internal energy and equation of state are given by corresponding expressions appropriate to an OCP. As an application of this finding, the equation of state for such a relatively high Z plasma is obtained from a previously constructed expression for the Helmholtz free energy [9].

With the plasma under consideration comprised of electrons and ions of charge eZ, charge neutrality implies the constraint

$$eV(n_e - Z\, n_Z) \simeq 0 \qquad (1)$$

where V is plasma volume and n_e and n_Z are electron and ion number densities, respectively. The approximate equality in

(1) derives from the inherent statistical nature of a plasma. For a two-component plasma, three interactions contribute to the mean two-particle potential energy and we write

$$\langle V\rangle \simeq (4\pi)^{1/3}[n_e^{1/3}e^2 - (n_e n_Z)^{1/6}Ze^2 + n_Z^{1/3}(Ze)^2] \qquad (2)$$

With the constraint (1) we may set $n_e = Z\,n_Z$ and (2) becomes

$$\langle V\rangle \simeq e^2(4\pi n_Z)^{1/3}(Z^{1/3} - Z^{7/6} + Z^2) \qquad (3)$$

Thus in the limit

$$Z^2 >> Z^{1.17} \qquad (4)$$

the relation (3) reduces to

$$\langle V\rangle \simeq (4\pi n_Z)^{1/3}(Ze)^2 \qquad (5)$$

which we recognize to be the interaction potential of a one-component plasma comprised of ions of charge Ze.

This similarity may be further illustrated through the plasma parameter [10,11]

$$\gamma^{2/3} \simeq \frac{\langle V\rangle}{\langle E_K\rangle} \simeq \frac{\langle V\rangle}{k_B T} \qquad (6)$$

where $\langle E_K\rangle$ is mean kinetic energy per particle. The relevance of γ to the properties of a plasma is evident from (6). Namely, with this expression we may conclude that a plasma is strongly coupled when $\gamma \gtrsim 1$ and weakly coupled when $\gamma << 1$.

Substituting (5) into (6) gives (dropping the Z subscript on n_Z)

$$\gamma^{2/3} \simeq \frac{(Ze)^2(4\pi n)^{1/3}}{k_B T} \tag{7}$$

which, again, is the plasma parameter relevant to an OCP with ions of charge Ze.

The canonical expression for γ is given by

$$\gamma = \frac{1}{4\pi n\lambda_D^3} \tag{8}$$

where λ_D is the Debye distance.[13,14] With the latter two expressions we find

$$\lambda_D^{\,2} = \frac{k_B T}{4\pi n(Ze)^2} \tag{9}$$

which is seen to be the Debye distance for an OCP comprised of ions of charge Ze.

These relations may be cast in terms of a plasma parameter more common to studies of OCP. It is given by

$$\Gamma = (Ze)^2/ak_B T \tag{10}$$

where a^3 is a measure of the mean occupation volume per ion. That is,

$$\frac{4}{3}\pi a^3 n = 1 \tag{11}$$

The parallel structure of γ and Γ is evidenced by rewriting (8):

$$\gamma = \frac{(Ze)^2/\lambda_D}{k_B T} \tag{12a}$$

$$\Gamma = \frac{(Ze)^2/a}{k_B T} \tag{12b}$$

so that

$$\gamma^2 = 3\,\Gamma^3$$

We may conclude that the previously stated criterion that separates weakly from strongly coupled plasmas may also be given in terms of Γ.

Thus, we find that in the limit (4), the coupling and parameters of a two-component plasma of electrons and ions of charge Ze reduce to those relevant to an OCP of ions of charge Ze in a negative background. We may conclude that thermodynamic properties such as internal energy and equation of state for a two-component laboratory plasma with $Z \gg Z^{1.17}$ are the same as those of an OCP comprised of the same species of ions.

Numerical work of Slattery, Doolan and DeWitt [9] established the following expression for internal energy U for the fluid phase of an OCP.

$$\frac{U}{Nk_B T} = \bar{a}\Gamma + b\Gamma^{1/4} + c\Gamma^{-1/4} + d + \bar{e}\Gamma/N \tag{13}$$

where N is total ion number and

$\bar{a} = -0.898$, $b = 0.950$, $c = 0.190$,

$d = -0.815$, $\bar{e} = 0.010$

The Helmholtz free energy, F, may then be obtained through integration. Namely,

$$\frac{F(\Gamma)}{Nk_BT} = \int_{\Gamma_1}^{\Gamma} \left[\frac{U(\Gamma')}{Nk_BT} + 3\right] d\Gamma' + \frac{F(\Gamma_1)}{Nk_BT} \tag{14}$$

The normalized free energy $F(\Gamma_1)/Nk_BT$, with $\Gamma_1 = 1$ was calculated employing various contributions over the unit interval. There results (in the limit of large N) [9]

$$\frac{F(\Gamma)}{Nk_BT} = \bar{a}\Gamma + 4(b\Gamma^{1/4} - c\Gamma^{-1/4}) + (d+3)\ln\Gamma - (\bar{a} + 4b - 4c + 1.152) \tag{15}$$

With this value of free energy at hand an equation of state is obtained from the thermodynamic relation

$$P = -\left(\frac{\partial F}{\partial V}\right)_T \tag{16}$$

To perform this differentiation we first rewrite Γ (10,12b) in explicit form

$$\Gamma = \frac{(Ze)^2}{k_BT}\left(\frac{4\pi N}{3V}\right)^{1/3} \tag{17}$$

There results

$$\frac{\partial\Gamma}{\partial V} = -\frac{\Gamma}{3V} \tag{18}$$

Differentiating (15) we obtain

$$\frac{1}{Nk_BT}\frac{\partial F}{\partial V} = [\bar{a} + b\Gamma^{-3/4} + c\Gamma^{-5/4} + (d+3)\Gamma^{-1}]\frac{\partial\Gamma}{\partial V} \tag{19}$$

With (16) and (18) we then obtain

$$\frac{PV}{Nk_BT} = 1 + \frac{d}{3} + \frac{1}{3}\left(\bar{a}\Gamma + b\Gamma^{1/4} + c\Gamma^{-1/4}\right) \qquad (20)$$

With the previously stated argument we may conclude that (13) and (20) are valid energy and equation-of-state formulas for a laboratory plasma comprised of ions and electrons in equilibrium at a given temperature and obeying the constraint (4). Note in particular that for $Z = 5$ (completely ionized boron) $Z^{1.17}/Z^2 = 0.26$. For $Z = 13$ (completely ionized aluminum) this ratio becomes 0.12. Thus the proposed equivalence should be valid for $Z \gtrsim 5$. It is also important to note that this equivalence is not appropriate to processes where the dynamics of electrons come into play, such as conductivity [12].

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